, the larger m is, the larger $\sin\theta$ is. However, the maximum value that $\sin\theta$ can have is 1, for an angle of 90°. (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find which m corresponds to this maximum diffraction angle.

Solution

Solving the equation $d\sin\theta = m\lambda$ for m gives

$$m = \frac{d\sin\theta}{\lambda}. \tag{27.8}$$

Taking $\sin\theta = 1$ and substituting the values of d and λ from the preceding example gives

$$m = \frac{(0.0100\text{ mm})(1)}{633\text{ nm}} \approx 15.8. \tag{27.9}$$

Therefore, the largest integer m can be is 15, or

$$m = 15. \tag{27.10}$$

Discussion

The number of fringes depends on the wavelength and slit separation. The number of fringes will be very large for large slit separations. However, if the slit separation becomes much greater than the wavelength, the intensity of the interference pattern changes so that the screen has two bright lines cast by the slits, as expected when light behaves like a ray. We also note that the fringes get fainter further away from the center. Consequently, not all 15 fringes may be observable.

27.4 Multiple Slit Diffraction

An interesting thing happens if you pass light through a large number of evenly spaced parallel slits, called a **diffraction grating**. An interference pattern is created that is very similar to the one formed by a double slit (see Figure 27.16). A diffraction grating can be manufactured by scratching glass with a sharp tool in a number of precisely positioned parallel lines, with the untouched regions acting like slits. These can be photographically mass produced rather cheaply. Diffraction gratings work both for transmission of light, as in Figure 27.16, and for reflection of light, as on butterfly wings and the Australian opal in Figure 27.17 or the CD pictured in the opening photograph of this chapter, Figure 27.1. In addition to their use as novelty items, diffraction gratings are commonly used for spectroscopic dispersion and analysis of light. What makes them particularly useful is the fact that they form a sharper pattern than double slits do. That is, their bright regions are narrower and brighter, while their dark regions are darker. Figure 27.18 shows idealized graphs demonstrating the sharper pattern. Natural diffraction gratings occur in the feathers of certain birds. Tiny, finger-like structures in regular patterns act as reflection gratings, producing constructive interference that gives the feathers colors not solely due to their pigmentation. This is called iridescence.

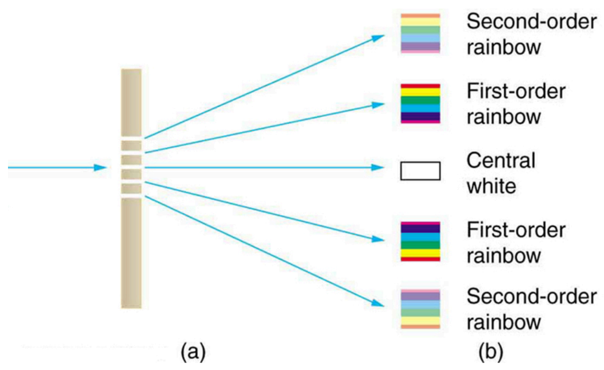

Figure 27.16 A diffraction grating is a large number of evenly spaced parallel slits. (a) Light passing through is diffracted in a pattern similar to a double slit, with bright regions at various angles. (b) The pattern obtained for white light incident on a grating. The central maximum is white, and the higher-order maxima disperse white light into a rainbow of colors.

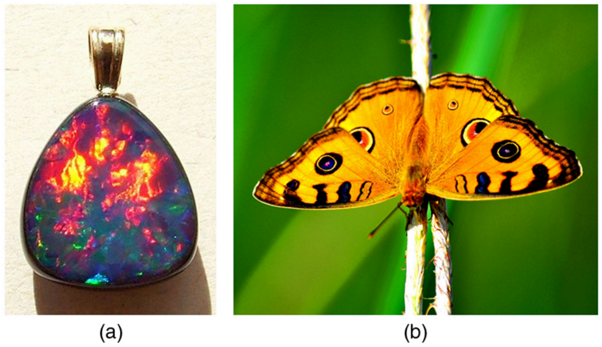

Figure 27.17 (a) This Australian opal and (b) the butterfly wings have rows of reflectors that act like reflection gratings, reflecting different colors at different angles. (credits: (a) Opals-On-Black.com, via Flickr (b) whologwhy, Flickr)

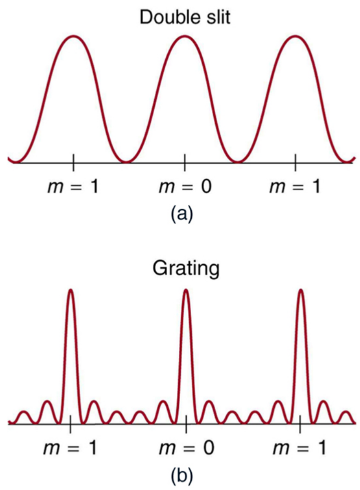

Figure 27.18 Idealized graphs of the intensity of light passing through a double slit (a) and a diffraction grating (b) for monochromatic light. Maxima can be produced at the same angles, but those for the diffraction grating are narrower and hence sharper. The maxima become narrower and the regions between darker as the number of slits is increased.

The analysis of a diffraction grating is very similar to that for a double slit (see Figure 27.19). As we know from our discussion of double slits in Young's Double Slit Experiment, light is diffracted by each slit and spreads out after passing through. Rays traveling in the same direction (at an angle θ relative to the incident direction) are shown in the figure. Each of these rays travels a different distance to a common point on a screen far away. The rays start in phase, and they can be in or out of phase when they reach a screen, depending on the difference in the path lengths traveled. As seen in the figure, each ray travels a distance $d \sin \theta$ different from that of its neighbor, where d is the distance between slits. If this distance equals an integral number of wavelengths, the rays all arrive in phase, and constructive interference (a maximum) is obtained. Thus, the condition necessary to obtain **constructive interference for a diffraction grating** is

$$d \sin \theta = m\lambda, \text{ for } m = 0, 1, -1, 2, -2, \ldots \text{ (constructive)}, \tag{27.11}$$

where d is the distance between slits in the grating, λ is the wavelength of light, and m is the order of the maximum. Note that this is exactly the same equation as for double slits separated by d. However, the slits are usually closer in diffraction gratings than in double slits, producing fewer maxima at larger angles.

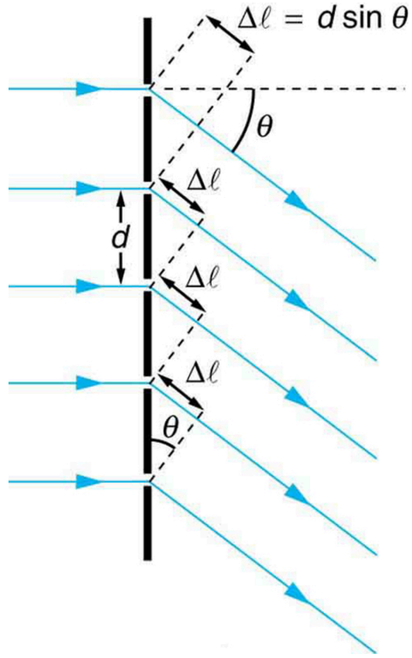

Figure 27.19 Diffraction grating showing light rays from each slit traveling in the same direction. Each ray travels a different distance to reach a common point on a screen (not shown). Each ray travels a distance $d \sin\theta$ different from that of its neighbor.

Where are diffraction gratings used? Diffraction gratings are key components of monochromators used, for example, in optical imaging of particular wavelengths from biological or medical samples. A diffraction grating can be chosen to specifically analyze a wavelength emitted by molecules in diseased cells in a biopsy sample or to help excite strategic molecules in the sample with a selected frequency of light. Another vital use is in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths. A range of diffraction gratings are available for selecting specific wavelengths for such use.

Take-Home Experiment: Rainbows on a CD

The spacing d of the grooves in a CD or DVD can be well determined by using a laser and the equation $d \sin\theta = m\lambda, \text{ for } m = 0, 1, -1, 2, -2, \ldots$. However, we can still make a good estimate of this spacing by using white light and the rainbow of colors that comes from the interference. Reflect sunlight from a CD onto a wall and use your best judgment of the location of a strongly diffracted color to find the separation d.

EXAMPLE 27.3

Calculating Typical Diffraction Grating Effects

Diffraction gratings with 10,000 lines per centimeter are readily available. Suppose you have one, and you send a beam of white light through it to a screen 2.00 m away. (a) Find the angles for the first-order diffraction of the shortest and longest wavelengths of visible light (380 and 760 nm). (b) What is the distance between the ends of the rainbow of visible light produced on the screen for first-order interference? (See Figure 27.20.)

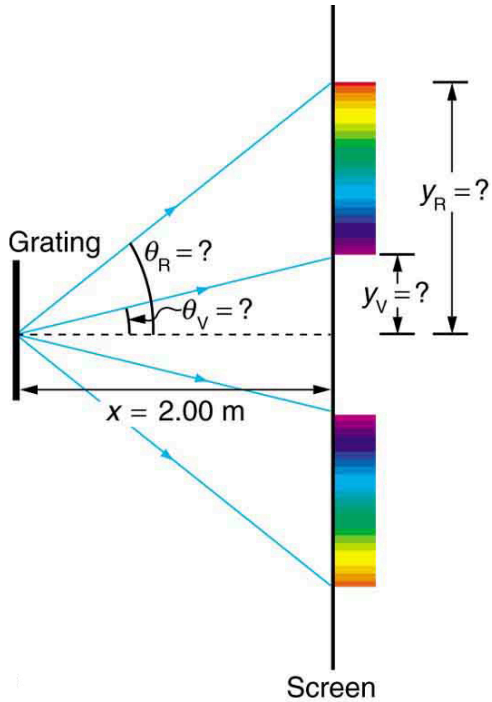

Figure 27.20 The diffraction grating considered in this example produces a rainbow of colors on a screen a distance $x = 2.00$ m from the grating. The distances along the screen are measured perpendicular to the x-direction. In other words, the rainbow pattern extends out of the page.

Strategy

The angles can be found using the equation

$$d \sin\theta = m\lambda \text{ (for } m = 0, 1, -1, 2, -2, \ldots) \tag{27.12}$$

once a value for the slit spacing d has been determined. Since there are 10,000 lines per centimeter, each line is separated by $1/10{,}000$ of a centimeter. Once the angles are found, the distances along the screen can be found using simple trigonometry.

Solution for (a)

The distance between slits is $d = (1 \text{ cm})/10{,}000 = 1.00 \times 10^{-4}$ cm or 1.00×10^{-6} m. Let us call the two angles θ_V for violet (380 nm) and θ_R for red (760 nm). Solving the equation $d \sin\theta_V = m\lambda$ for $\sin\theta_V$,

$$\sin\theta_V = \frac{m\lambda_V}{d}, \tag{27.13}$$

where $m = 1$ for first order and $\lambda_V = 380 \text{ nm} = 3.80 \times 10^{-7}$ m. Substituting these values gives

$$\sin\theta_V = \frac{3.80 \times 10^{-7} \text{ m}}{1.00 \times 10^{-6} \text{ m}} = 0.380. \tag{27.14}$$

Thus the angle θ_V is

$$\theta_V = \sin^{-1} 0.380 = 22.33^\circ. \tag{27.15}$$

Similarly,

$$\sin\theta_R = \frac{7.60 \times 10^{-7} \text{ m}}{1.00 \times 10^{-6} \text{ m}}. \tag{27.16}$$

Thus the angle θ_R is

$$\theta_R = \sin^{-1} 0.760 = 49.46^\circ. \tag{27.17}$$

Notice that in both equations, we reported the results of these intermediate calculations to four significant figures to use with the calculation in part (b).

Solution for (b)

The distances on the screen are labeled y_{V} and y_{R} in Figure 27.20. Noting that $\tan\theta = y/x$, we can solve for y_{V} and y_{R}. That is,

$$y_{\text{V}} = x\tan\theta_{\text{V}} = (2.00\text{ m})(\tan 22.33^{\circ}) = 0.815\text{ m} \tag{27.18}$$

and

$$y_{\text{R}} = x\tan\theta_{\text{R}} = (2.00\text{ m})(\tan 49.46^{\circ}) = 2.338\text{ m}. \tag{27.19}$$

The distance between them is therefore

$$y_{\text{R}} - y_{\text{V}} = 1.52\text{ m}. \tag{27.20}$$

Discussion

The large distance between the red and violet ends of the rainbow produced from the white light indicates the potential this diffraction grating has as a spectroscopic tool. The more it can spread out the wavelengths (greater dispersion), the more detail can be seen in a spectrum. This depends on the quality of the diffraction grating—it must be very precisely made in addition to having closely spaced lines.

27.5 Single Slit Diffraction

Light passing through a single slit forms a diffraction pattern somewhat different from those formed by double slits or diffraction gratings. Figure 27.21 shows a single slit diffraction pattern. Note that the central maximum is larger than those on either side, and that the intensity decreases rapidly on either side. In contrast, a diffraction grating produces evenly spaced lines that dim slowly on either side of center.

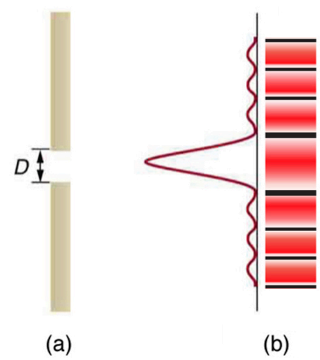

Figure 27.21 (a) Single slit diffraction pattern. Monochromatic light passing through a single slit has a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The drawing shows the bright central maximum and dimmer and thinner maxima on either side.

The analysis of single slit diffraction is illustrated in Figure 27.22. Here we consider light coming from different parts of the *same* slit. According to Huygens's principle, every part of the wavefront in the slit emits wavelets. These are like rays that start out in phase and head in all directions. (Each ray is perpendicular to the wavefront of a wavelet.) Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. When they travel straight ahead, as in Figure 27.22(a), they remain in phase, and a central maximum is obtained. However, when rays travel at an angle θ relative to the original direction of the beam, each travels a different distance to a common location, and they can arrive in or out of phase. In Figure 27.22(b), the ray from the bottom travels a distance of one wavelength λ farther than the ray from the top. Thus a ray from the center travels a distance $\lambda/2$ farther than the one on the left, arrives out of phase, and interferes destructively. A ray from slightly above the center and one from slightly above the bottom will also cancel one another. In fact, each ray from the slit will have another to interfere destructively, and a minimum in intensity will occur at this angle. There will be another minimum at the same angle to the right of the incident direction of the light.